**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with ** = 45 minutes and ** = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Answer : B**

The average time increase 45 minutes to 55 minutes because 10 min late service manager,So

** = 45

** = 8 minutes

z = (X-μ)/б

= (60-55)/8

= 0.625

1 - stats.norm.cdf(abs(0.625))

=**0.2659**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean ** = 38 and Standard deviation ** =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**False, Above 44**

1 – stats.norm.cdf(44,loc=38,scale=6) = 0.26598

**Between 44 and 38**

stats.norm.cdf(44,loc=38,scale=6)- stats.norm.cdf(38,loc=38,scale=6)

= **0.34134**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True**

**Employees under 30 yrs of age**

stats.norm.cdf(30,loc=38,scale=6) = **0.09121**

**No. of employees attending training program from 400 numbers**

400\*stats.norm.cdf(30,loc=38,scale=6) = **36.4844**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer** - As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ). **Therefore in the question 2X1~ N(2 u,4 σ^2) and X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 ) 2X1-(X1+X2) = N( 4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Answer - D**

stats.norm.interval(0.99,100,20) = **(48.4834 , 151.5165 )**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer:**

A)Total mean = mean1 + mean2

Mean = 7+5

In indain rupees

Mean\*45 = 540

Total standard deviation

sd = np.sqrt((9)+(16))

sd\*45 = 225.0

A Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

stats.norm.interval(0.95,540,225) = **(99.00810347848784, 980.9918965215122)**

**B)** Specify the 5th percentile of profit (in Rupees) for the company,

To compute 5th Percentile, we use formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X= 540+(-1.645)\*(225)

**np.round(X) = 170.0**

**C)** Probability of Division 1 making a loss P(X<0)

stats**.**norm**.**cdf(0,5,3) = 0.04779

*Probability of Division 2 making a loss P(X<0)*

stats**.**norm**.**cdf(0,7,4) = **0.040059**

**END!**